## Exercise 1

1. We have defined our function “midpointquad.m” with using given syntax from the exercise. By filling up the gaps in the code line on the syntax we have obtained our function as follow; 
2. After defining the function, we have tested it with . We know that result of the function “midpointquad.m” should be exact, as the given function has degree just . After executed the following code, we have obtained the expected output;  
   
3. After make sure that our function works correctly, we have used the function to estimate Runge function , on the interval and obtained the following results,

|  |  | Midpoint Result | Error |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

1. In order to estimate the order of accuracy, we have looked at the ratio of the error at the last two steps. The ratio was approximately , which means . Since we were multiplying with 10, and the ratio is , we have that the order of accuracy .

## Exercise 2

1. We have estimated given integrals for each function below by using our midpointquad function with , on the interval , to test the exactness and obtained the following results;

| Function | Midpoint Result | Error |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

1. We can see from the result that the degree of the exactness of midpoint quadrature rule is just , meaning that the rule is exact for any polynomial with degree .
2. We know that from the first exercise, the order of accuracy of Midpoint Quadrature rule , which means that the degree of exactness is on less than the order of accuracy. Therefore, the midpoint rule is also satisfying this special case.

## Exercise 3

1. We have defined our function “trapezoidquad.m” with using the formula that we have proven in the class. As a different part from the midpoint rule, in this case we have used a for loop while evaluating the function for the given data points. It also can be done just like in midpoint function, but interestingly that definition syntax raised some errors during execution, therefore we have used the loop technique as follow; 
2. After defining the function, and make sure that it works correctly, we have estimated given integrals for each function below by using our new trapezoid function with , on the interval , to test the exactness and obtained the following results;

| Function | Midpoint Result | Error |
| --- | --- | --- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

1. We can see from the result that the degree of the exactness of trapezoid quadrature rule is just , meaning that the rule is exact for any polynomial with degree , just like the midpoint quadrature rule.
2. We have used the trapezoid function to estimate Runge function , on the interval and obtained the following results,

|  |  | Midpoint Result | Error |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

1. In order to estimate the order of accuracy, we have looked at the ratio of the error at the last two steps. The ratio was approximately , which means . Since we were multiplying with 10, and the ratio is , we have that the order of accuracy , just like in the Midpoint-Rule, as we excepted.
2. We have seen that from the previous exercise, the order of accuracy of Trapezoid Quadrature Rule , and the degree of exactness of it is just , which means that the degree of exactness is on less than the order of accuracy. Therefore, the midpoint rule is also satisfying this special case, just like the Midpoint Quadrature Rule.

## Exercise 4

1. In order to estimate the integral , we have applied Midpoint Quadrature rule on the interval with following subintervals. We cannot apply Trapezoidal Rule here, as the logarithm function does not define at . Also, by the asymptotic convergent formula, we can see that as goes to goes to . This means that the convergent rate will be less than usual cases. Let us see the results from the table;

|  |  | Midpoint Result | Error |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

1. In order to estimate the order of accuracy, we have looked at the ratio of the error at the last two steps. The ratio was approximately , which means . Since we were multiplying with 10, and the ratio is , we have that the order of accuracy , it is less than usual cases as we excepted because the asymptotic convergent rate goes to near the end point .

## Exercise 5

2) We wrote a function for calculating the quadrature given by the NC formula, for a function over an interval [a, b], with N many evenly spaced points in this interval. We called it nc\_single.m.

8) Using the function we wrote for 2x with the command *nc\_single(@(x) 2\*x,0,1,2)* we observe that indeed the quadrature gives us 1 as the result.

| Function | Error ( |xTrue - xApprox| ) | | |
| --- | --- | --- | --- |
| N=4 | N=5 | N=6 |
| 4 \* x^3 | 2.2204e-16 | 0 | 2.2204e-16 |
| 5 \* x^4 | 0.0185 | 0 | 2.2204e-16 |
| 6 \* x^5 | 0.0556 | 0 | 0 |
| 7 \* x^6 | 0.1091 | 0.0026 | 0.0015 |
| Degree | 3 | 5 | 5 |

## Exercise 6

| n | nc\_single Result | Error |
| --- | --- | --- |
| 3 | 6.794871794871794 | 4.048070260981762 |
| 7 | 3.870448673470800 | 1.123647139580768 |
| 11 | 4.673300555653498 | 1.926499021763466 |
| 15 | 7.89954464085154 | 5.152743106961507 |

Results in the table showed us that raising in a Newton-Cotes rule is not the way to get increasing accuracy as we can confirm it with the error column in the table.

## Exercise 7

7) We get the same result in nc\_quad.m as well.

8) We made our routine check by evaluating the nc\_single function on the newly defined function partly\_quadratic with N = 3 and 2 number of subintervals. When we have 2 subintervals, which means one of them corresponds to the (-1,0) and the other one (0,1), our evaluation will be exact for each part of the given subintervals from the definition of the Newton-Cotest method. Therefore, the result should be nothing but 1/6, which exactly matches the output that we obtain from the MatLAB.

9) After doing the routine check by evaluating the nc\_single function on the newly defined function partly\_quadratic with N = 3 and 2 number of subintervals, now we would like to make the same test with just incresaing the number of subintervals that we use, namely we let it to be 3. When we have 3 subintervals, our evaluation will not be exact anymore because right now the middle subinterval will be a function that is not differantiable at the point 0 so we have a non-smooth function and therefore the obtained example points for the method will not be exact anymore. Thus the overall resulting of the function should be different than 1/6, which exactly matches the output that we obtain from the MatLAB.



10) The solution exists in the diary file.

12)

| Subintervals | N | nc\_quad Error | Error ratio |
| --- | --- | --- | --- |
| 10 | 2 | 2.7561 | 0.9964 |
| 20 | 2 | 2.7462 | 1.0001 |
| 40 | 2 | 2.7466 | 1.0001 |
| 80 | 2 | 2.7468 | 1 |
| 160 | 2 | 2.7468 | 1 |
| 320 | 2 | 2.7468 |  |
| 10 | 3 | 2.7429 | 1.0014 |
| 20 | 3 | 2.7468 | 1 |
| 40 | 3 | 2.7468 | 1 |
| 80 | 3 | 2.7468 | 1 |
| 160 | 3 | 2.7468 | 1 |
| 320 | 3 | 2.7468 |  |
| 10 | 4 | 2.7453 | 1.0005 |
| 20 | 4 | 2.7468 | 1 |
| 40 | 4 | 2.7468 | 1 |
| 80 | 4 | 2.7468 | 1 |
| 160 | 4 | 2.7468 | 1 |
| 320 | 4 | 2.7468 |  |

## Exercise 8

1) We wrote a function for calculating the quadrature given by the Gauss-Legendre formula, for a function over an interval [a, b], with N many evenly spaced points in this interval.

Using the function we wrote for 2x with the command *gl\_single(@(x) 2\*x,0,1,1)* we observe that indeed the quadrature gives us 1 as the result.

2)

| f | Error N=2 | Error N=3 |
| --- | --- | --- |
| 3 \* x^2 | 0 | 0 |
| 4 \* x^3 | 1.1102e-16 | 0 |
| 5 \* x^4 | 0.0278 | 0 |
| 6 \* x^5 | 0.0833 | -2.2204e-16 |
| 7 \* x^6 | 0.1574 | 0.0025 |
| Degree | 3 | 5 |

3)

| N | gl\_single Result | Error |
| --- | --- | --- |
| 3 | 4.7917 | 2.0449 |
| 7 | 3.0806 | 0.3338 |
| 11 | 2.8123 | 0.0655 |
| 15 | 2.7601 | 0.0133 |

Results in the table showed us that raising in a Gauss-Legendre quadrature

rule is increasing the accuracy. We can confirm it with the error column in the table.